Closed-Economy, One-Period Model

Chapter 5, Part 2

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When Government Expenditures Have a Purpose

- For simplicity, we assumed that government expenditures had no effect on the utility
- In reality, governments spend on things people appreciate:
  - Roads
  - Parks
  - National defense
  - Communal amenities
- Government expenses are not (necessarily) wasteful!

When Government Expenditures Have a Purpose

- Let's assume now, that households appreciate government expenses
- This would change the consumers' utility function:

$$U(C,l) \quad \rightarrow \quad U(C,l,\mathbf{G})$$

- Notice that households **do not** choose G.

Consumers' Problem

- Households still maximize their utility, limited by the usual constraints:

$$\max_{C,l} U(C, l, G)$$
$$\text{subject to } \; C + wl = wh + \pi - T$$

- In this case, **only** the utility function changes.

Firms' Problem

- What is the problem of the firm in this case?

Let's Solve Now for a Competitive Equilibrium

- Production function:
$$Y = 10N$$
- Utility function:
$$u(C, l) = \log(C + 0.1G) + \log(l)$$
- Time constraint:
$$l = 1 - N^s$$
- Government spends G, and taxes T

Firm's problem

- Firm's objective:

$$\max_{N} 10N - wN$$

- Firm's optimal decision:

$$w = 10$$

- Firm prodices output as long the wage is at least 10.

- Profit:

$$\pi(w) = Y - wN = 10N - wN = 0$$

Household's problem

- Household's optimal decision:

$$w = \frac{C + 0.1G}{1 - N} \qquad \text{and} \qquad C = wN + \pi - T$$

- Solving for labor supply:

$$N^s(w) = \frac{1}{2} + \frac{T - \pi - 0.1G}{2w}$$

Equilibrium

In equilibrium w = 10, π = 0 and T = G

- Labor:

$$N^d = N^s = \frac{1}{2} + \frac{G - 0 - 0.1G}{20} = \frac{1}{2} + 0.045G$$

- Output:

$$Y = 10N = 5 + 0.45G$$

- Consumption

$$C = Y - G = 5 - 0.55G$$

Crowding Out When Government Expenditure is Useful

- Increase in G has a smaller effect on output, compared to wasteful G
- When consumers appreciate G, the marginal utility of consumption is **lower**, for each level of consumption
- This makes them **more willing** to increase leisure, and work less

Do Households Prefer More, or Less Government?

In our example:

- Leisure:

$$l = \frac{1}{2} - 0.045G$$

- Consumption:

$$C = Y - G = 5 - 0.55G$$

- Utility

$$U(C, l) = \log(5 - 0.55G) + \log(0.5 - 0.045G)$$

What happens when G increases?

In-class Assignment:

Solve for the competitive equilibrium with this setting:

- Production function:

$$Y = 10N$$

- Utility function:

$$u(C, l) = \log(C + \mathbf{1.1G}) + \log(l)$$

- Time constraint:

$$l = 1 - N^s$$

- Government spends G, and taxes T

Do households prefer more, or less G?

Distiortionary Taxation

- So far we only talked about lump-sum taxation
  - Taxes that people pay don't depend on their actions
- However, lump-sum taxes almost do not exist
  - In reality taxes are function of economic choices, like income
- Because these taxes are function of choices, they also **affect** choices. That is why we call them **distortionary**

Let's Study A Simple Model

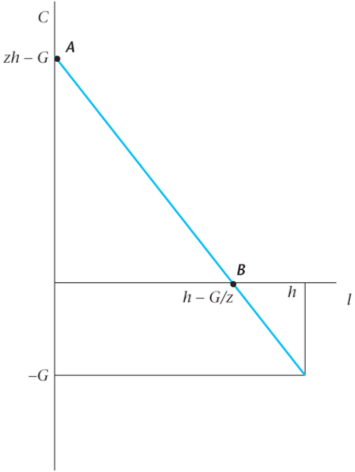
- Use the model to study the incentive effects of the income tax rate
- We'll also derive the **Laffer curve**
- We assume labor is the only input, with constant returns to scale (linear production function)

$$Y = z \cdot N^d$$

Production Possibiliyt Frontier

Production Possibilities Frontier has a very simple form

$$C = z \cdot (h - l) - G$$



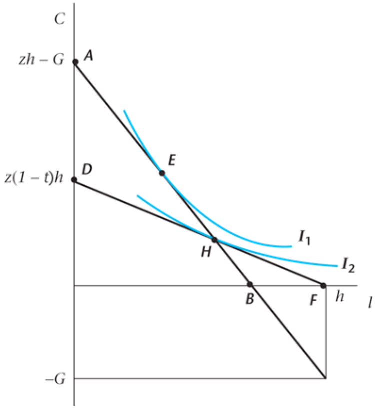
Consumer's budget constraint

- Consumers pay a flat rate tax on the wage income they earn

$$C = (1 - t) \cdot w \cdot (h - l) + \pi$$

- What is the slope of budget constraint?
- What is the slope of the production possibility frontier?

Flat rate taxes are distortionary



- The equilibrium point is H
- The slope of the budget constraint is not equal to the slope of production possibility frontier
- This is an inefficient equilibrium!
- At H, $MRT_{l,C} > MRS_{l,C}$

Profits for the firm

- Firm's profits are

$$\pi = Y - w \cdot N^d = (z - w) \cdot N^d$$

- In equilibrium $w = z$, hence $\pi = 0$

- Therefore, the consumer's budget constraint can be simplified

$$C = (1 - t) \cdot w \cdot (h - l)$$

Government's Revenue

- With lump-sum taxes, the government's revenue is

$$Rev = T$$

- With a proportional tax rate t, it's revenue is

$$Rev(t) \quad = \quad t \cdot w \cdot N(t) \quad = \quad t \cdot z \cdot \Big(h - l(t) \Big)$$

where $l(t)$ is the household' leisure choice <u>when the tax rate is t</u>

- We want to study how government's revenue changes with the tax rate t

Government's Revenue

Changing t has **two** effects on revenue:

- On one hand, increasing the tax rate increases revenue

$$Rev(t) = \mathbf{t} \cdot w \cdot \Big(h - l(t)\Big)$$

- On the other hand, if the tax rate causes people to work less

$$Rev(t) = t \cdot w \cdot \Big(h - \mathbf{l(t)}\Big)$$

- The **Laffer curve** is relationship between the tax rate and its revenue

Illustrative Example: Laffer Curve

- Production function: $Y = 4N$ (no capital)
- Utility function: $u(C, l) = \sqrt{C} + 2\sqrt{l}$ and $l = 1 - N^s$
- Government expenditure is G and tax rate is t. Note that a balanced government budget requires

$$G = t \cdot w \cdot N$$

- Marginal product of labor: $MP_N = 4$
- Marginal rate of substitution: $MRS_{l,C} = \frac{2\sqrt{C}}{\sqrt{l}} = \frac{2\sqrt{C}}{\sqrt{1-N}}$

Illustrative Example: Laffer Curve

- From the firms' optimality we know $w = 4$, and that profits are zero
- From the household problem we know that

$$\frac{2\sqrt{C}}{\sqrt{1-N}} = w \cdot (1-t) \quad \text{and} \quad C = (1-t) \cdot w \cdot N$$

- Replace $C = (1-t) \cdot N$ and solve for N

$$N = \frac{w \cdot (1-t)}{4 + w \cdot (1-t)}$$

Illustrative Example: Equilibrium

- Therefore the labor supply as function of t is

$$N = \frac{1-t}{2-t}$$

- Government revenue

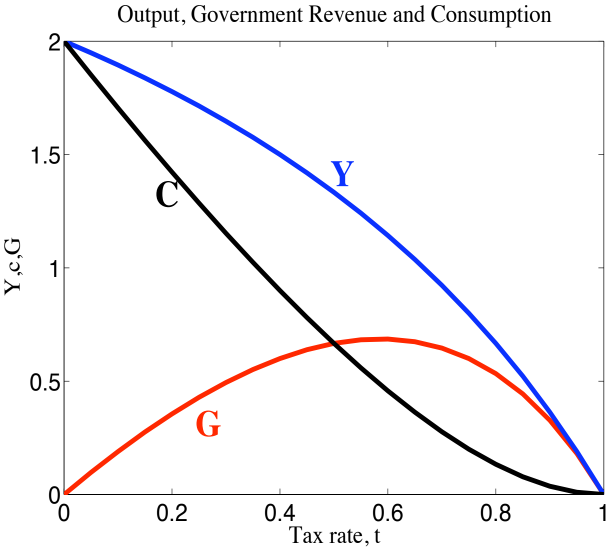
$$Rev(t) = t \cdot w \cdot N(t) = t \cdot 4 \cdot \frac{(1-t)}{2-t} = G$$

- Output and consumption

$$Y = 4\frac{(1-t)}{2-t}$$

$$C = Y - G = 4\frac{(1-t)^2}{2-t}$$

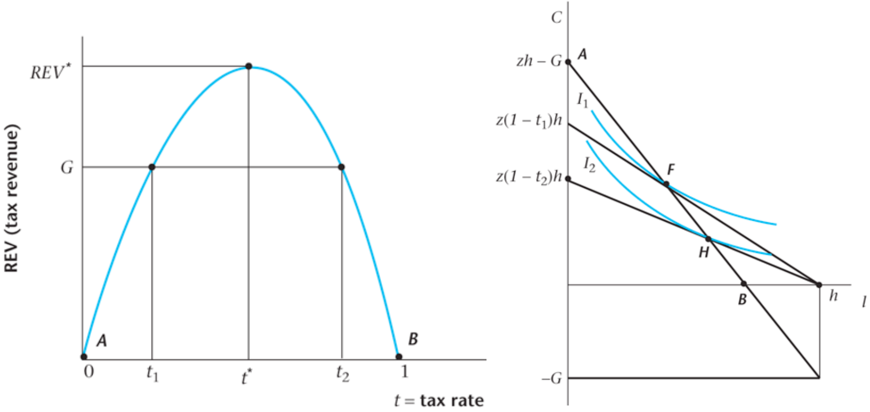
Example: Equilibrium



What do we observe?

- Government revenue as a function of tax rate is **hump-shaped**: it first increases and then decreases
- If the tax rate is too high, the government can increase revenue by **lowering** the tax rate
- If $t = 0$, there's no revenue; same if $t = 1$
- Every level of revenues has two tax rates able to collect as much
- Output and consumption decrease as taxes increase

Laffer Curve and Two equilibria



Which tax rate is preferred to finance an expenditure level G?

Why Do Americans Work So Much More Than Europeans?

- Prescott, in his 2004 Quaterly Review tries to answer this question
- In the 70s, Europeans worked more than in the US
- In the 90s, this pattern reversed.
- What happened in Europe? **Taxes went up**

Hours Worked and Output per Hour: Europe vs U.S.

Period	Country	Output per person 15-64	Hours worked per person	Output per hour worked
1993-96	Germany	74	75	99
	France	74	68	110
	Italy	57	64	90
	Canada	79	88	89
	U.K.	67	88	76
	Japan	78	104	74
	U.S.	100	100	100
1970-74	Germany	75	105	72
	France	77	105	74
	Italy	53	82	65
	Canada	86	94	91
	U.K.	68	110	62
	Japan	62	127	49
	U.S.	100	100	100

Source: Prescott (Quaterly Review, 2004): "Why Do Americans Work So Much More Than Europeans?"

Hours Worked and Tax Rate: Europe vs U.S.

Period	Country	Hours per person 15-64 per week	Effective tax rate
1993-96	Germany	19.3	0.59
	France	17.5	0.59
	Italy	16.5	0.64
	Canada	22.9	0.52
	U.K.	22.8	0.44
	Japan	27	0.37
	U.S.	25.9	0.4
1970-74	Germany	24.6	0.52
	France	24.4	0.49
	Italy	19.2	0.41
	Canada	22.2	0.44
	U.K.	25.9	0.45
	Japan	29.8	0.25
	U.S.	23.5	0.4

Observations

- During 1993-96:
  - Labor supply is much higher in the US and Japan
  - US output per person is about 40 percent higher than europeans
  - Most of this difference is due to hours worked and not productivity

- During 1970-74
  - Europeans worked more than Americans
  - Their productivity was lower
  - Without changes in their productivities, their output would have been much higher if they didn't decrease working hours

- Tax rates have gone up in Europe and stayed the same in the US

Why Do Americans Work So Much More Than Europeans?

- Prescott uses a model similar to ours
- He calibrates his model to match facts in the US data
- Next, he replaces the US tax rates with the European tax rates
- He finds after this chance in taxes, agents in his model work less
- He estimates that two 2/3 of the differences in GDP per person between US and Europe can be accounted for by higher taxes in Europe
- These taxes cause people to work less
- The answer to the question: tax rates are higher in Europe

Take Home Points

- Flat rate taxes distort household marginal decision
- There are **welfare losses** due to this distortion
- There are also output losses because of this distortion
- When government expenditures have to be financed through flat rate taxes, increases in G do not increase output and labor